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Stacking Sequence and curing Temperature Effect on Natural Frequency of Hybrid fiber Reinforced Composite Laminate

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Abstract: In this work, a multi-lamina hybrid fiber reinforced structure was statically and harmonically analyzed for the purpose of studying the effect of stacking sequence, curing limit upon induced stresses and natural frequency of free longitudinal in-plane vibrations. The effect of boundary conditions of the structure on the natural frequency under free longitudinal vibration due to the tension was also studied. The laminate was suggested to be composed of 4 layers and subjected to a tensile force with thermal load represented by curing the structure at a temperature of 240 oC, then it is cooled to a temperature of 23℃. Two stacking layouts are suggested (0o/90o /0o/90o and 0o/90o/90o/0o) in order to investigate their effects on the natural frequency. The software of FEA ANSYS v.14 is taken to manipulate the project data. A comparison between numerical results obtained from the software and theoretical ones obtained from the analytical solution based on Generalized Hooke's Law and classical lamination theory was made for the purpose of results verification. Good convergence was found between the two sets of results referred to above.

Keywords: Stacking, sequence, Curing, temperature, Boundary Condition, Natural, frequency, Hybrid composite Laminate.

1 INTRODUCTION:

In many engineering and industrial applications plate structures are widely used and the vibration problems became very critical and sensitive in dynamic applications since the probability of occurrence of resonance is likely to happen. It is very important to make these structures as thin as possible for the purposes of total weight and cost, but for the purpose of strength and durability, it is essential to have some extra thickness. In addition, the fundamental frequency is decreased as the thickness gets reduced, so to compromise on these controversial issues, it is resorted to fiber reinforced composite plate structures. The various schemes of structures reinforcing by continuous fibers or other methods are normally recommended and approved for various high-specific-strength and -stiffness applications with a minimally safe thickness. Since they are exposed to fail due to resonance under different excitations. Composites and hybrid composites substitute materials in various industrial and engineering structures such as aero-space, land- and marine-applications, special-purpose and pressure tanks for their specific functional requirements and characteristics. The mechanical behavior of such structures is widely different from those isotropic counterparts due to their orthotropicity of constituting laminates. To avert the damage of such entities arised from undesired-harmful vibrations and resonance, it will be so necessary to define and compute the resonant-natural frequencies of a structure, such that spectrum, transient and frequency analyses include modal analysis must be carried out, where the natural frequencies of them can be found [1]. The mode shapes must also be determined in order to strengthen the maximally critical sections or to specify suitable locations where it is necessary to minimize part weighing or to maximize dampening [2]. Sahu and Asha (2005) [2,3], used an 8-noded-isoparametric element of quadratic shell type to achieve a finite element analysis, in order to study the stability and response of pre-twisted-panels, in addition to the effect of various geometrical parameters such as the twist-angle, aspect ratio, variable factors of lamination, shallowness ratio . Tita (2003) [3] worked on the theoretical and experimental dynamic analysis of E-glass reinforced epoxy resin. He used [0/90] S and [±45] S laminates in his work. The laminates were fabricated by hand-lay-up process and cut to beam shape specimens. The specimens were used as free end beams in the vibration measurements. He calculated the mechanical properties of the composite analytically and used them in his simulations of the dynamic properties. He presented his ANSYS® simulation results in contour format showing the mode types and shapes. His experimental results on vibration are in a graphical format demonstrating the frequency response of the two laminates. Colakoglu performed vibration experiments on 10-layer beam specimens of glass polyethylene composite at a range of temperatures. The vibration was induced by the impact of spherical steel ball hammer [4]. ANSYS® numerical simulations were also used to obtain the frequency response. Teng and Hu (2001) [5] discussed the designing variables and effective factors for constrained-lamina-damping structures using Ross-Kerwin-Ungar (RKU) model.

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They also investigated the effects of temperature, frequency and damped structures dimensions on vibration damping features and properties. Nayak et al. during 2002 prepared a higher order theory for the vibration characteristics of the laminated plate [5]. Chao presented free vibration natural frequencies of rectangular plates [6]. Jaehong prepared in 2002 a generally-analytical-model which can be appropriately applied to the dynamic response of a thin wall channel composite section [7]. Ajay introduced an analysis depending on a higher-order shear deformation theory for the free vibration behavior of sandwiched-skew laminates [8]. Jun, L. et al. studied the free vibration and buckling behaviors of axially loaded laminated composite beams having arbitrary lay-up using the dynamic stiffness method [9]. Mehmet ÇEVIK discussed the in-plane-free vibration behavior of symmetric angular-ply laminated composite curved beams and performed it using finite-element-method. The effects of rotary inertia and shear deformation were considered in the discussion. The curved members with opening angle (α) from 30° up to 270°are considered. Studies related to various affecting factors were achieved to investigate the influences of angle of fiber inclination, boundary conditions, orthotropicity of material properties, (radius / width) ratio and layers number upon resonant-natural-frequency [10]. In a torsional loading mode of traditional and classical composite materials, the free vibration analysis have also been widely conducted, such that finding of free torsional-vibrations-properties, of composite laminate is considered one of important criteria for design and configuration of parts in engineering and industry. Mohammed Fahmy Aly (2013) studied and investigated the twisted vibrations of composite multi-layers laminates of doubly symmetric cross-sections analytically using theory of classical-lamination (CLT) while, the coupling of bending and twisting deformation caused by inclination of fibers of the laminates was ignored. The researcher also investigated twisting vibrations of the laminates depending on shear-deformation-theory where the shearing distortion influences were accounted for [11].The applications where the torsion and bending loading are combined in a part are also investigated by M. L. Pavan Kishore and R. K. Behera (2015). A typical impulse-propeller constructed from conventional composites was produced and its vibration features and properties were analyzed. Optimal design of a fiber-reinforced-composite impulse-propeller was achieved for different restricted and unrestricted designing purposes. Only symmetric-ply stacking layouts were considered. Results show that the ply stacking sequence has an effect on the characteristics of a conventional propeller. Proper stacking sequence of the composite propeller improves its performance as compared to its metallic counterpart [12]. For hybrid fiber reinforced composites, Omar A. Mohammed studied the effect of number of carbon layer, position and orientation angle of the laminate on the natural frequency and mode shape for hybrid fiber (carbon / glass) with epoxy composite laminates only. Numerical analyses were used to study vibration behavior of composite laminated beams using ANSYS 13 software. The results showed that the natural frequencies increased when the number of carbon layer increases and decreased when the carbon layer position changes from the surface towards mid-plane, also; the natural frequencies change with changing orientation angle [13]. If the exciting frequency is kept rather moderate, through increasing the structural-natural frequency, the probability of resonant damage occurrence could then be minimized. In accordance with that, some of the experts and specialists during the last three decades of last century concentrated on increasing the lowest harmonic frequency of laminated structures. They studied the effect of various boundary conditions and loading schemes on the maximum fundamental frequency of angle ply composite laminates [14]. In this work unlikewise to previous literatures mentioned above, a thermo-mechanical analysis is made considering a hybrid fiber reinforced composite plate structure composed of 4 layers in a crossed ply laminated structure. Boundary conditions, curing temperature and stacking sequence are taken as the study parameters and their effect on the natural frequency is also investigated. The problem is solved through the finite element technique by the package of ANSYS v.14 using element type of three dimensional layered shell element 4node181as shown in Fig.1. This element is adopted since it permits introducing and saving the properties of all layers of a composite structure and it has six degrees of freedom including in- and out of plane linearly displaced deformation. The elastic properties of the hybridized materials are calculated using MATLAB v.2011.

2 Finite Element Modeling of the Hybrid Composite Plate Structure

The geometry of the hybrid laminated composite plate structure is illustrated in Figure 2. The multilayer unidirectional laminated composite plate structure consisting of 4 to 6 layers from hybrid fiber reinforced composites composed of short (chopped) boron fibers embedded in a matrix material of epoxy, polyester, polyamide and polyethylene to form together a composite matrix. The latter then is reinforced with long continuous parallel fibers of the same type as the chopped ones for the purpose of economizing. The plate is 400-mms. long, 20-mms wide and of 4.8 mms thick. The structure is then meshed using three dimensional layered structural shell element 4node181 as shown in Fig.1.

3 Mathematic Formulation of the Problem and Model Elastic Properties

Under bending the controlling relevant vibration differential formulas can be obtained from the buckling differential equations by adding an acceleration term to the right-hand part of the like-equilibrium-formulas. As with both plate bending and buckling problems, plate vibrations include coupling between bending-twisting when the plate is unsymmetrically laminated. For symmetrically laminated plates, the coupling vanishes. The governing differential equations of such a problem are as following [15, 16]:

$$\delta N_{x,\ x} + \delta N_{xy,\ y} = 0 \quad (1)$$

$$\delta N_{xy,\ x} + \delta N_{y,\ y} = 0 \quad (2)$$

$$\delta M_{x,\ xx} + 2\delta M_{xy,\ xy} + \delta M_{y,\ yy} + \overline{N}_x\, \delta W_{,\ xx} + 2\overline{N}_{xy}\delta W_{,\ xy} + \overline{N}_y\, \delta W_{,\ yy} = \rho\, \delta W_{,\ tt} \quad (3)$$

When the coupling vanishes, the vibration case could then be converted to the solution's form of Eq.3, since the rotary inertia effects are ignored. There is another possibility of a different type of coupling which is the bending-extension coupling which is more likely to happen in our model. The classical lamination theory (CLPT) is adopted to be the basis on which the problem is manipulated. The classic theorem of laminated plates whose acronym (CLPT) depends upon on Kirchhoff's presumptions, assuming that transversely normal with shearing stresses throughout the cross-section of plates are ignored. In elaboration this theorem, some extra presumptions are accounted for as under [17]:

1. The layers are perfectly bonded together (assumption).
2. The material of each layer is linearly elastic and has three planes of material symmetry (i.e., orthotropic).Each layer is of uniform thickness.
3. The strains and displacements are small.
4. The transverse shear stresses on the top and bottom surfaces of the laminate are zero.

Considering the above assumption the displacement field of CLPT is of form:

$$u(x, y, z) = u_0\,(x, y) - z\,\frac{\partial w_0}{\partial x}$$
$$v(x, y, z) = v_0\,(x, y) - z\,\frac{\partial w_0}{\partial y} \quad (4)$$
$$w(x, y, z) = w_0\,(x, y)$$

Where (uo, vo, wo) are the linear displacement-constituents throughout the (x-y-z) Cartesian-coordinate axes, seriesly, of a point on the middle plane (at z equals zero). The displacement-field requires that straight lines perpendicular to x-y-plane prior to distorsion are still straight-normal to the middle surface afterwards (beyond deformation). In the derivation process, the undermentioned essential presumptions were made:

1. The plate is constructed of an arbitrarily chosen magnitude of orthotropic plies assembled and joined together. The principal-axes of material of each single ply need not to geometrically conform to plate axes.
2. The plate is thin and has a constant thickness; i.e. the thickness h is much smaller than other dimensions.
3. The inplane displacements u, v in x and y directions, respectively, and the transverse displacement w in the z-direction are all small compared to the plate thickness.
4. Inplane strains εx; εy, and γxy are small compared to unity.
5. Each ply obeys Hooke's law; linear elastic behavior.

4 General Vibrational Analysis

Free vibration means the motion of a structure without any dynamic external forces, moments or support motion. The general equation of motion of an undamped linearly-SDOF-structures can be referred to as [18]:

$$\boldsymbol{m}\frac{d^2u}{dt^2} + \boldsymbol{ku} = \boldsymbol{0} \quad (5)$$

Free vibration can be commenced by exciting a structure at its static-equilibrium-status. By giving the mass certain an initial-displacement u (0) and initial-velocity $\dot{\boldsymbol{u}}$(0), at a zerol-time, characterized as the instant- motion is commenced:

$$u = u\,(0), \quad \dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}(0) \quad (6)$$

Solving of Eq.5 is obtained by standard methods as:

$$u(t) = u(0)\cos \omega_n t + \frac{\dot{u}\,(0)}{\omega_n}\, sin\, \omega_n\, t \quad (7)$$

Where natural-circular-frequency of oscillation expressed in units of (rad / sec) is given by:

$$\boldsymbol{\omega_n} = \sqrt{\frac{\boldsymbol{k}}{\boldsymbol{m}}} \quad (8)$$

The duration taken for a non-damped entity to achieve a single-cycle of free-vibration is the natural time-period of oscillation of that assemblage.

$$T_n = \frac{\boldsymbol{2\pi}}{\boldsymbol{\omega_n}} \quad (9)$$

Natural-cyclic-frequency of oscillation (vibration) is symbolized as ($\boldsymbol{f_n} = \frac{\boldsymbol{1}}{\boldsymbol{T_n}}$), unit in Hz (cycle / sec).

4.1 Mode Shape Formulation

The solution of an Eigen value problem can result in the natural frequencies and mode shape of an assembly. The set free-vibration of a non-damped structure in one of its natural-vibration-modes can be represented by the following expression:

$$u(t) = q_n\,(t)\,\varphi_n \quad (10)$$

Where, φ_n is not a time-dependent parameter. The time-alteration of the displacements can be represented by the following simple-harmonic-equation (SHE):

$$q_n(t) = A_n cos\omega_n t + B_n \sin \omega_n\, t \quad (11)$$

An, Bn are constants of integration. Combining above two equations results in:

$$u(t) = \varphi_n\,(A_n cos\omega_n t + B_n \sin \omega_n\, t) \quad (12)$$

Putting in equation of undamped free vibration, we have:

$$[-\,\omega_n^2\, m\, \varphi_n + k\, \varphi_n]q_n(t) = 0 \quad (13)$$

Either, $q_n(t)$= 0, ⇒ u(t)= 0 , trivial solution Or,

$$[k - m\omega_n^2]\,\varphi_n = 0 \quad (14)$$

A group of 'n' algebraic -homogeneous equations is for that 'n' no. of element. This set has always the trivial solution $\varphi_n = 0$, it implies no motion. The nontrivial solution is:

$\det[\boldsymbol{k} - \boldsymbol{m}\boldsymbol{\omega}_n^2] = \mathbf{0}$, This is called frequency equation. (15)
It gives N roots in ω_n^2 determine N natural frequencies. The roots are called Eigen values or normal values., there are N independent vectors (φ_n) which are called natural mode shapes of vibration corresponds to the N natural vibration frequencies ω_n of an N-DOF System, Eigen vectors or normal modes i.e. ({φ}i – Eigenvector) representing the mode shape of the ith natural frequency. It is possible for the Eigen vectors to be found from the following matrix-form-expression [19]:

$$|[\boldsymbol{D}] - \boldsymbol{\lambda}\,[\boldsymbol{I}]| = \mathbf{0} \quad (16)$$

Where $[D]$ = Dynamic matrix, or often called materials properties matrix $[k]$ = Stiffness matrix, $[M]$ = Mass matrix. Such that:
$[D]$= $[M]$-1 $[k]$, Eigen vector $\lambda = \omega$2.
The element stiffness matrix can be expressed as:

$$[k] = \iint [B]\text{T}. [D]. [B]\ \text{dx.dy} \quad (17)$$

Where: [B] is the standard strain- displacement matrix. The consistent mass matrix is expressed as:

$$[M] = \iint [N]\text{T}. [P]. [N]\ \text{dx.dy} \quad (18)$$

Where: [N] is the shape function matrix in (ξ, η, ζ coordinates) prescribed in terms of non-dependent parameters such as (x, y, etc.), and the term [P] is often referred to as the stress divergence or stress force term such that [19], [20]:

$$u = [N].\vec{q} \quad , \quad \varepsilon = [B].].\vec{q} \quad \text{and} \quad \sigma = [d].[B].\vec{q} \quad (19)$$

4.2 Analyzing of Natural-Frequency of a Laminate:

4.2.1 Displacement:

In a more specific discipline, the controlling differential expression for free-vibration of symmetric laminates in accordance with the theorem of classical lamination in terms of the dynamic equilibrium of the infinitesimal element shown in Fig. 3 yields the following partial differential equation of motion (neglecting both shear deformation and rotary inertia) [21]:

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2\,(D_{12} + 2\,D_{66})\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = -\rho\frac{\partial^2 w}{\partial t^2} \quad (20)$$

In the derivation process, the undermentioned essential presumptions has been made:

1. The plate is constructed of an arbitrarily chosen sum of orthotropic plies assembled together by an adhesive. The principal axes of a certain single ply material needn't to be matched with geometrical coordinate-axes of the plate.
2. The plate is thin and has a constant thickness; i.e. the thickness h is much smaller than other dimensions.
3. The inplane displacements u, v in x and y directions, respectively, and the transverse displacement w in the z direction are all small compared to the plate thickness.
4. Inplane strains εx; εy, and εxy are small compared to unity.
5. Each ply obeys Hooke's law; linear elastic behavior. Where D11, D12, D22, and D66 are rigidities in the principal materials axis-direction, and ρ is the averaged mass-density of all laminates.

4.2.2 Stress-Strain Relationships and Equation of Motion

The total strains can be given as [17], [22]:

$$\begin{bmatrix}\boldsymbol{\epsilon}_{xx}\\ \boldsymbol{\epsilon}_{yy}\\ \boldsymbol{\gamma}_{xy}\end{bmatrix} = \begin{bmatrix}\boldsymbol{\epsilon}_{xx}^{0}\\ \boldsymbol{\epsilon}_{yy}^{0}\\ \boldsymbol{\gamma}_{xy}^{0}\end{bmatrix} + \boldsymbol{z} * \begin{bmatrix}\boldsymbol{\epsilon}_{xx}^{1}\\ \boldsymbol{\epsilon}_{yy}^{1}\\ \boldsymbol{\gamma}_{xy}^{1}\end{bmatrix} \quad (21)$$

$$\begin{bmatrix}\boldsymbol{\epsilon}_{xx}\\ \boldsymbol{\epsilon}_{yy}\\ \boldsymbol{\gamma}_{xy}\end{bmatrix} = \begin{bmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial y}\\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\end{bmatrix} + z * \begin{bmatrix}-\frac{\partial^2 w_0}{\partial x^2}\\ -\frac{\partial^2 w_0}{\partial y^2}\\ -2 * \frac{\partial^2 w_0}{\partial x \partial y}\end{bmatrix} \quad (22)$$

Where $(\boldsymbol{\epsilon}_{xx}^{0}, \boldsymbol{\epsilon}_{yy}^{0}, \boldsymbol{\gamma}_{xy}^{0})$ are the membrane strains and $(\boldsymbol{\epsilon}_{xx}^{1}, \boldsymbol{\epsilon}_{yy}^{1}, \boldsymbol{\gamma}_{xy}^{1})$ are the flexural (bending) strains, known as the curvatures [23]. The transformed stress-strain relations of an orthotropic lamina in a plane state of stress are; for $\bar{Q}_{ij}$ can be given as:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k \begin{Bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} \quad (23)$$

The resultant of in-plane forces Nxx, Nyy and Nxy and moments Mxx, Myy and Mxy applied on a laminated structure can be mathematically found by stress-integration method in every ply or layer through the laminated structure-thickness. Using the stress in terms of the displacement, we can obtain the in-plane force resultants Nxx, Nyy, Nxy, and moments Mxx, Myy and Mxy. The in-plane force resultants can be given as:

$$\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix} = \sum_{k=1}^{N} \int_{z=k}^{z=k+1} \begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k dz \quad (24)$$

Where σ_{xx} , σ_{yy} and σ_{xy} are the normal and shear stresses.

$$\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\boldsymbol{\epsilon}_{xx}^{0}\\ \boldsymbol{\epsilon}_{yy}^{0}\\ \boldsymbol{\gamma}_{xy}^{0}\end{bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}\boldsymbol{\epsilon}_{xx}^{1}\\ \boldsymbol{\epsilon}_{yy}^{1}\\ \boldsymbol{\gamma}_{xy}^{1}\end{bmatrix} \quad (25)$$

While the moments (Mxx, Myy and Mxy) are calculated as following:

$$\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix} = \sum_{k=1}^{N} \int_{z=k}^{z=k+1} \begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k z.dz \quad (26)$$

$$\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix} = \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{bmatrix}\boldsymbol{\epsilon}_{xx}^{0}\\ \boldsymbol{\epsilon}_{yy}^{0}\\ \boldsymbol{\gamma}_{xy}^{0}\end{bmatrix} + \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{bmatrix}\boldsymbol{\epsilon}_{xx}^{1}\\ \boldsymbol{\epsilon}_{yy}^{1}\\ \boldsymbol{\gamma}_{xy}^{1}\end{bmatrix} \quad (27)$$

Where Aij are the extensional strain stiffness, Bij the coupling stiffness, and Dij the bending stiffness. These stiffnesses are determined as below [17], [23]:

$$A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_{k+1} - z_k) \quad (28)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_{k+1}^2 - z_k^2) \quad (29)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_{k+1}^3 - z_k^3) \quad (30)$$

When the temperature effect is encountered, the stress-strain relations will be put in the following forms [24]:

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} - \alpha_{xx} T_0 \\ \frac{\partial v_0}{\partial y} - \alpha_{yy} T_0 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - \alpha_{xy} T_0 \end{bmatrix} + z * \begin{bmatrix} -\frac{\partial^2 w_0}{\partial x^2} - \alpha_{xx} T_1 \\ -\frac{\partial^2 w_0}{\partial y^2} - \alpha_{yy} T_1 \\ -2 * \frac{\partial^2 w_0}{\partial x \partial y} - \alpha_{xy} T_1 \end{bmatrix} \quad (31)$$

For uniform linear temperature change: $\Delta T = T_0 (x, y, T) - zT_1 (x, y, T)$, thus Eq. 31 becomes:

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} - \alpha_{xx} \Delta T \\ \frac{\partial v_0}{\partial y} - \alpha_{yy} \Delta T \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - \alpha_{xy} \Delta T \end{bmatrix} + z * \begin{bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2 * \frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix} \quad (32)$$

Where α_{xx}, α_{yy} *and* α_{xy} are the thermal expansion coefficients defined as:

$$\alpha_{xx} = \alpha_{11} \cos^2\theta + \alpha_{22} \sin^2\theta$$
$$\alpha_{yy} = \alpha_{11} \sin^2\theta + \alpha_{22} \cos^2\theta$$
$$2\alpha_{xy} = 2(\alpha_{11} - \alpha_{22}) \sin\theta \cos\theta \quad (33)$$

Where α_{11} *and* α_{22} are the longitudinal and transverse thermal expansion coefficients respectively and θ is the lamination angle. The change in temperature can be defined as:

ΔT = applied temperature – reference temperature

Where reference temperature T_{ref} = 23 °C

The transformed stress-strain relationships of an orthotropic lamina in a plane state of stress in terms of $\bar{Q}_{ij}$ and including temperature effect are put in the form of [15], [17], [23]:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \epsilon_{xx} - \alpha_{xx} \Delta T \\ \epsilon_{yy} - \alpha_{yy} \Delta T \\ \gamma_{xy} - 2 \alpha_{xy} \Delta T \end{Bmatrix} \quad (34)$$

When accounting for determination of thermal and bending stresses N^t *and* M^t respectively such that [23], [24]:

$$\begin{Bmatrix} N_{xx}^t, M_{xx}^t \\ N_{yy}^t, M_{yy}^t \\ N_{xy}^t, M_{xy}^t \end{Bmatrix} = \sum_{k=1}^{n} \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 2\alpha_{xy} \end{Bmatrix} (1, z) \Delta T \, dz \quad (35)$$

Then, the equation of motion including the thermal effects is got to be:

$$\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{Bmatrix} \ddot{u}_0 \\ \ddot{v}_0 \\ \ddot{w}_0 \end{Bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q \end{Bmatrix} + \begin{Bmatrix} f_1^t \\ f_2^t \\ f_3^t \end{Bmatrix} \quad (36)$$

Where Cij terms are the stiffness Matrix terms of the composite materials.

and $f_1^t = \frac{\partial N_{xx}^t}{\partial x} + \frac{\partial N_{xy}^t}{\partial y}$, $f_2^t = \frac{\partial N_{xy}^t}{\partial x} + \frac{\partial N_{yy}^t}{\partial y}$ *and* $f_3^t = -\left(\frac{\partial^2 N_{xx}^t}{\partial x^2} + 2 \frac{\partial^2 N_{xy}^t}{\partial x \partial y} + \frac{\partial^2 N_{yy}^t}{\partial y^2}\right)$ (37)

5 Elastic Properties Determination

The hybrid composite materials used in this research are composed of a composite matrix constituted from a typical matrix such as epoxy or polystyrene reinforced with short fibers randomly distribute throughout as shown in Fig. 4. Thus this composite matrix displays an isotropic elastic behavior. The composite matrix is then reinforced with long, parallel and equally spaced fibers to form the hybrid fiber reinforced composite lamina as shown in Fig.4 which represents the constituent unit of the whole laminate structure of interest. Let (E1m and E2m) be the longitudinal and transverse moduli for unidirectional fiber 0° composite matrix shown in Fig.3 of the same fiber aspect ratio and fiber volume fraction as the randomly oriented discontinuous fiber matrix, so [25], [26]:

$$E_{1m} = \frac{1 + 2.a_f . \eta_l . \forall_{sfm}}{1 - \eta_l . \forall_{sfm}} . E_m \quad (38)$$

$$E_{2m} = \frac{1 + 2 . \eta_T . \forall_{sfm}}{1 - \eta_T . \forall_{sfm}} . E_m \quad (39)$$

$$G_{12m} = \frac{1 + \eta_G . \forall_{sfm}}{1 - \eta_G . \forall_{sfm}} . G_m \quad (40)$$

$$\nu_{12m} = \nu_{sf} . \forall_{sfm} + \nu_m . \forall_{mm} \quad (41)$$

Where:

$$\eta_l = \frac{\frac{E_{sf}}{E_m} - 1}{\frac{E_{sf}}{E_m} + 2.a_f} \quad (42)$$

$$\eta_T = \frac{\frac{E_{sf}}{E_m} - 1}{\frac{E_{sf}}{E_m} + 2} \quad (43)$$

$$\eta_G = \frac{\frac{G_{sf}}{G_m} - 1}{\frac{G_{sf}}{G_m} + 1} \quad (44)$$

The terms E1m, E2m, G12m and ν_{12m} are the basic elements to determine the elastic constants of the composite matrix namely [26]:

$$E_{cm} = \frac{3}{8} . E_{1m} + \frac{5}{8} . E_{2m} \quad (45)$$

$$G_{cm} = \frac{1}{8} . E_{1m} + \frac{1}{4} . E_{2m} \quad (46)$$

Since the composite matrix is assumed to behave isotropically as referred to above, then the Poisson's ratio of the composite matrix (v_{cm}) can be given as:

$$v_{cm} = \left(\frac{E_{cm}}{2.G_{cm}} - 1\right) \quad (47)$$

Thus for the whole hybrid composite lamina shown in Fig.5, the overall elastic properties E1, E2, G12 and ν_{12} are derived from the first principles given in the relevant references to be given respectively as [27]:

$$E_1 = E_f.\forall_f + (1-\forall_f).E_m\left[\left(\frac{3.(1-\forall_f)+6.a_f.\eta_l.\forall_{sfp}}{8(1-\forall_f)-8\eta_l.\forall_{sfp}}\right) + \left(\frac{5.(1-\forall_f)+10.\eta_T.\forall_{sfp}}{8(1-\forall_f)-8\eta_T.\forall_{sfp}}\right)\right] \quad (48)$$

$$E_2 = \frac{E_f.E_{cm}\left[\left(\frac{3.(1-\forall_f)+6.a_f.\eta_l.\forall_{sfp}}{8(1-\forall_f)-8\eta_l.\forall_{sfp}}\right)+\left(\frac{5.(1-\forall_f)+10.\eta_T.\forall_{sfp}}{8(1-\forall_f)-8\eta_T.\forall_{sfp}}\right)\right]}{E_f(1-\forall_f)+E_{cm}.\forall_f\left[\left(\frac{3.(1-\forall_f)+6.a_f.\eta_l.\forall sfp}{8(1-\forall_f)-8\eta_l.\forall sfp}\right)+\left(\frac{5.(1-\forall_f)+10.\eta_T.\forall_{sfp}}{8(1-\forall_f)-8\eta_T.\forall_{sfp}}\right)\right]} \quad (49)$$

$$G_{12} = \frac{G_f.E_m.\left[\left(\frac{(1-\forall_f)+2.a_f.\eta_l.\forall_{sfp}}{(1-\forall_f)-\eta_l.\forall_{sfp}}\right)+\left(\frac{2.(1-\forall_f)+4.\eta_T.\forall_{sfp}}{(1-\forall_f)-\eta_T.\forall_{sfp}}\right)\right]}{8.G_f.(1-\forall_f)+E_m.\forall_f\left[\left(\frac{(1-\forall_f)+2.af.\eta_l.\forall_{sfp}}{(1-\forall_f)-\eta_l.\forall_{sfp}}\right)+\left(\frac{2.(1-\forall_f)+4.\eta_T.\forall_{sfp}}{(1-\forall_f)-\eta_T.\forall_{sfp}}\right)\right]} \quad (50)$$

$$v_{12} = v_f.\forall_f + \left(\frac{E_{cm}}{2.G_{cm}} - 1\right)(1-\forall_f) = v_f.\forall_f + v_{cm}.(1-\forall_f) \quad (51)$$

And

$$v_{21} = \frac{E_2}{E_1} v_{12} \quad (52)$$

Where:

E_{1m}: Longitudinal moduli for a unidirectional discontinuous fiber 00 composite matrix, combined of resin and discontinuous fiber.
E_{2m}: Transverse moduli for a unidirectional discontinuous fiber 00 composite matrix, combined of resin and discontinuous fiber.
E_{cm}: Moduli of isotropic composite matrix, combined of resin and random discontinuous fiber.
E_1: Longitudinal modulus for unidirectional continuous fiber 0° composite lamina, combined of composite matrix and continuous fiber.
E_2: Transverse modulus for unidirectional continuous fiber 0° composite lamina, combined of composite matrix and continuous fiber.
E_{sf}: Moduli of discontinuous fiber material.
E_f: Moduli of continuous fiber material.
E_m: Moduli of resin material.
G_{12m}: Shear modulus for a unidirectional discontinuous fiber 0° composite matrix.
G_{cm}: Shear modulus of isotropic composite matrix.
G_{12}: Shear modulus for a unidirectional continuous fiber 00 composite lamina.
G_{sf}: Shear modulus of discontinuous fiber material.
G_f: Shear modulus of continuous fiber material.
G_m: Shear of resin material.
ν_{12m}: The major Poisson's ratio for a unidirectional discontinuous fiber 00 composite matrix.
νcm: Poisson's ratio of isotropic composite matrix.
ν_{12}: The major Poisson's ratio for a unidirectional continuous fiber 0° composite lamina.
ν_{sf}: Poisson's ratio for discontinuous fiber material.
ν_f: Poisson's ratio for continuous fiber.
ν_m: Poisson's ratio for resin material.
$\forall_{sfp}$: Volume fraction of discontinuous fiber, ratio of the volume of discontinuous fiber to the volume of composite lamina.
$\forall_{mm}$: Volume fraction of resin matrix, ratio of the volume of resin to the volume of composite matrix.
$\forall_{mp}$: Volume fraction of resin matrix, ratio of the volume of resin to the volume of composite lamina.
$\forall_f$: Volume fraction of continuous fiber, ratio of the volume of continuous fiber to the volume of composite lamina.
$\forall_m$: Volume fraction of matrix, ratio of the volume of composite matrix to the volume of composite lamina.
a_f: The ratio of average fiber length to fiber diameter $= l_f / d_f$
d_f: Fiber diameter.
l_f : Average fiber length.
The aspect ratio of the long fibers considered in the lamina of interest cab be taken as (500 / 1) as a recommend value [28], [29].

6 Problem Characterization and Manipulation

The hybrid composite laminates considered in this work are symmetric and anti-symmetric cross-plies composed of 4 layers and subjected to a tensile force 3000N with thermal load represented by curing the structure at temperatures ranging from 240–195°C, then it is cooled to a temperature of 23°C, thus there is a residual stresses will be induced. The fibers of outer layers are oriented along the x-axis and those of inner layers are oriented along the y-axis. The Generalized Hooke's Law of stress-strain relationship states that [17]:

$$\sigma_i = C_{ij}.\epsilon_j, \quad i \geq 1, \ j \leq 6 \quad (53)$$

Or the strain can be given as:

$$\epsilon_j = S_{ij}.\sigma_i \quad (54)$$

Where Cij and Sij are the stiffness and compliance matrices respectively. For an orthotropic material having 3 mutual perpendicular axes of symmetry, the properties vary according to direction, thus Hooke's Law for such a material can be minimized to:

$$\begin{bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}\frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0\\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0\\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}\end{bmatrix} * \begin{bmatrix}\sigma_1\\ \sigma_3\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{bmatrix} \quad (55)$$

The material symmetry of orthotropic materials requires that:

Sij = Sji , Thus $\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$ (56)

Additionally, if it is assumed that the fibers bear load in the z direction and are isotropic, then all properties in the z-direction are equal to the properties in the x-direction. It is possible to derive the following relations [17]:

$E_1 = E_3 \ , \ \nu_{12} = \nu_{32} \ , G_{12} = G_{23} = G_{31} \ and \ \nu_{23} = \frac{E_2}{E_3} \nu_{32}$ (57)

These relationships are important when input material properties into ANSYS. Since the stiffness matrix must be positive definite, it will be assumed plane stress and plane strain conditions {all '3' terms (which containing "3" are neglected}. Then in the Generalized Hooke's Law for the case of plane stress, Cij is replaced by Qij, where $i \geq 1, \ j \leq 3$ such that:

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{E_1}{1-\nu_{12}*\nu_{21}}$$

$$Q_{12} = \frac{S_{12}}{S_{11}S_{22} - S_{12}^2} = \frac{\nu_{12}E_2}{1-\nu_{12}*\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}*\nu_{21}}$$

$$Q_{12} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_2}{1-\nu_{12}*\nu_{21}}$$

$$Q_{33} = \frac{1}{S_{66}} = G_{12} \quad (58)$$

Since there are both thermal and tensile loads therefore:

$$\tilde{N} = N_T + N_x$$
$$\acute{M} = M_T + 0 \quad (59)$$

Where Ḿ are the bending moments caused due to the coupling of extension and bending. Combining thermal and tensile effects, the equation for mid surface strains becomes:

$$\begin{Bmatrix} \epsilon^0 \\ -- \\ \kappa \end{Bmatrix} = \begin{Bmatrix} A' & B' \\ -- & -- \\ B' & D' \end{Bmatrix} \begin{Bmatrix} \tilde{N} \\ -- \\ \acute{M} \end{Bmatrix} \quad (60)$$

Where A' is the compliance due to Q, B' includes coupling effects, and D' terms include flexural rigidity for bending. Since the laminate is symmetric, no bending occurs.

Thus, $\acute{M} = B' = \kappa = 0$, then:

$$\{\epsilon^0\} = \{A'\}\{\tilde{N}\} \quad (61)$$

Since Ḿ can be neglected, D' does not need to be calculated. To calculate A', Aij should be inverted as following:

$$A_{ij} = \sum [Q]_k \, t_k \quad (62)$$

Since this composite was cured at 220 oC but is sitting in a room at 23℃. Thus the residual stresses resulting from the temperature difference must be considered. The temperature difference is calculated as under:

$$\Delta T = T_{ref} - T_{cur} \quad (63)$$

The thermal loads are expressed as:

$$\{N_T\} = \sum [Q]_k \{\alpha\}_k \, t_k \, \Delta T \quad (64)$$

But the stresses in each lamina, k, can be given as:

$$\{\sigma\}_k = [Q]_k (\{\epsilon^0\} - \{\alpha\}_k \Delta T) \quad (65)$$

7 Thermal Expansion Coefficients of the Laminate

It is still finally very important to determine the coefficients of thermal expansion (CTE) namely the longitudinal and transverse coefficients $\alpha_1 \ and \ \alpha_2$ respectively of the whole laminate, since they are dependent on fiber volume fractions of the constituents and their corresponding coefficients. It is found in the relevant literatures that these coefficients can be determined as [30], [31, [32]:

$$\alpha_1 = \frac{E_f V_f \alpha_f + E_m V_m \alpha_m}{E_1} \quad \text{and}$$

$$\alpha_2 = \alpha_m V_m (1 + \nu_m) + \alpha_f V_f (1 + \nu_f) - \nu_{12} \alpha_1 \quad (66)$$

Where: $E_f, \ V_f, \ \alpha_f, \ \ E_m, V_m \ and \ \alpha_m$ are the modulus of elasticity, volume fraction and CTE of fibers and matrix respectively. The values of CTE of fibers and matrix are taken to be (4.95e-6 m/m/℃ for boron fiber and 60E-6 m/m/℃ for epoxy matrix) [33]. Values of ν_f and ν_m, the Poisson's ratios of boron fiber and epoxy matrix are (0.2 and 0.35 respectively) [32].

8 Results and Discussion

from the performed analysis, it can be easily shown that the normal stresses (both longitudinal and transverse) induced are directly proportional to both curing temperature and short fibers volume fraction percentage in composite matrix for both symmetric and anti-symmetric structures at constant external load. this can be attributed to the fact of the increment in curing temperature leads to increasing of thermal residual stresses with temperature rise. from the other hand, increasing of short fiber volume fraction leads to the increment of total elastic strength and shear rigidity of the hybrid composite material resulting in maximizing of the material resistance to the external applied loads. this is also explained referred to by the definition of the stresses set up in a material exposed to external or internal loads. figs. 6 through 9 display the changes of these normal stresses for both structures, but it is seen that in anti-symmetric layout, these stresses are lower than those in the symmetric laminate and their changes are also less in divergence than their counterparts in the symmetric scheme. this is of course due to its lower resistance and larger response to the external applied loads as it is obviously shown through the comparison of the longitudinal displacement of both structures for the same fiber volume fraction and curing temperature range as listed in table-1. figs. 6 and 7 show the longitudinal and transverse normal stresses of the symmetric layout while figs. 8 and 9 show the longitudinal and transverse normal stresses of anti- symmetric layout. the natural frequencies are unaffected by the curing temperature variation even if there are some residual stresses left in the hybrid composite material, but they are remarkably affected by the change of short fibers volume fraction since the latter directly affects both the total mass and elastic moduli of the structure, this fact is clearly displayed through figs. 10 and

11 for both symmetric and anti-symmetric stacking systems. these natural frequencies are sensitive to type of stacking system, such that the natural frequencies of symmetric stacking are higher than their corresponding values of anti-symmetric laminate. This is due to the fact that the overall structural stiffness of symmetric layout is greater than that of anti-symmetric one.

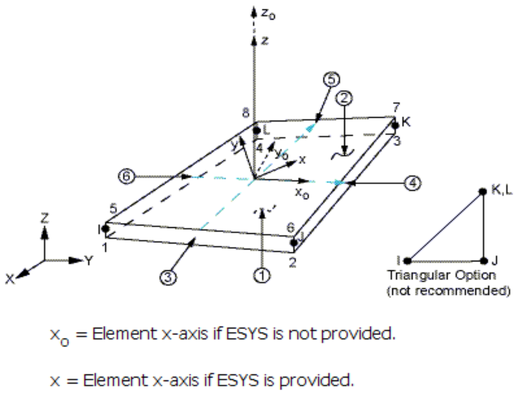

Fig.1: Three dimensional layered structural shell element 4node181.

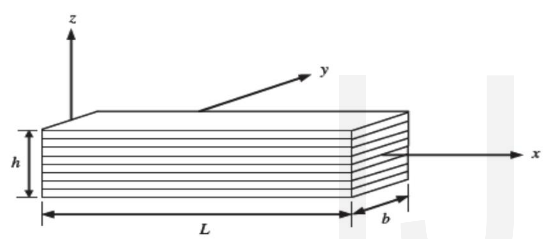

Fig.2: Hybrid laminated composite plate structure.

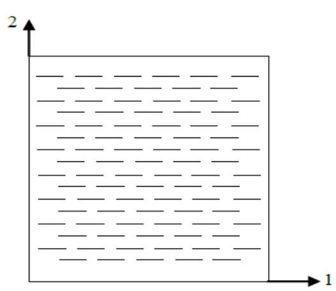

Fig. 3: unidirectional 0° short fibers composite matrix.

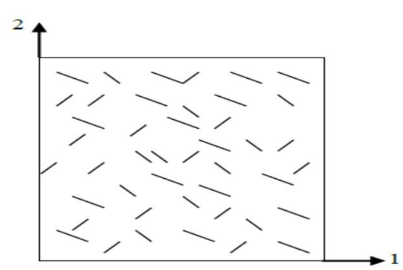

Fig.4: Randomly oriented discontinuous fiber of the composite matrix.

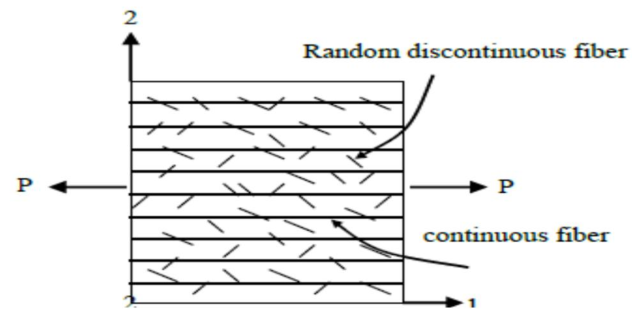

Fig.5: The hybrid fiber reinforced composite lamina.

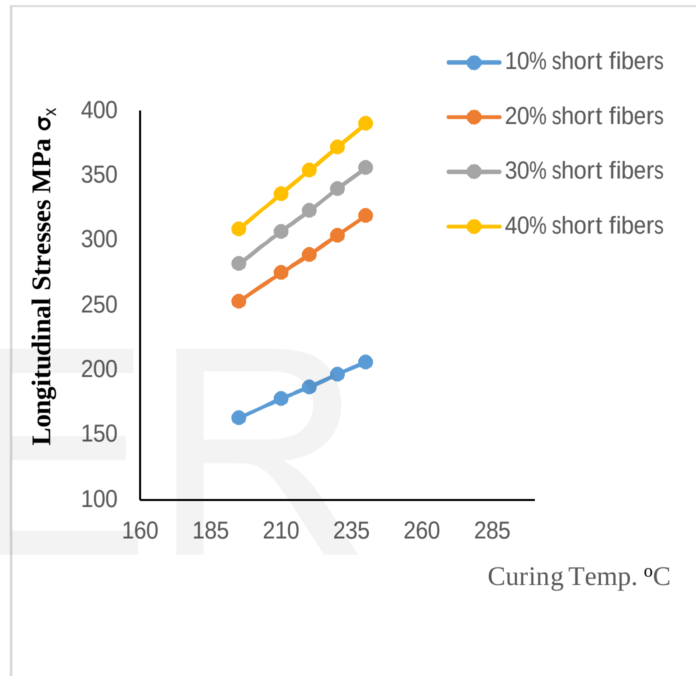

Fig.6: Effect of curing temp. on the longitudinal stresses (σx) of symmetric stacking sequence.

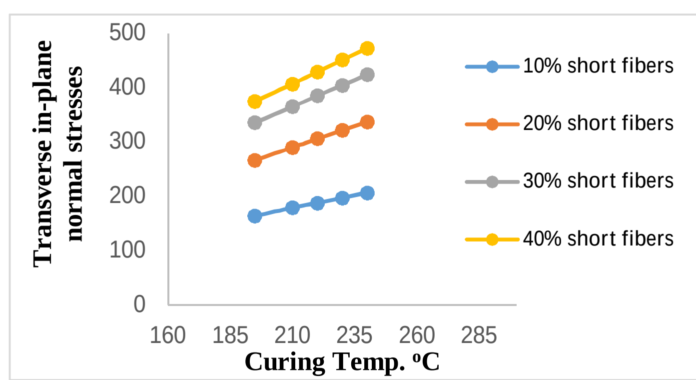

Fig.7: Effect of curing temp. on the transverse in-plane normal stresses (σy) of a symmetric stacking sequence.

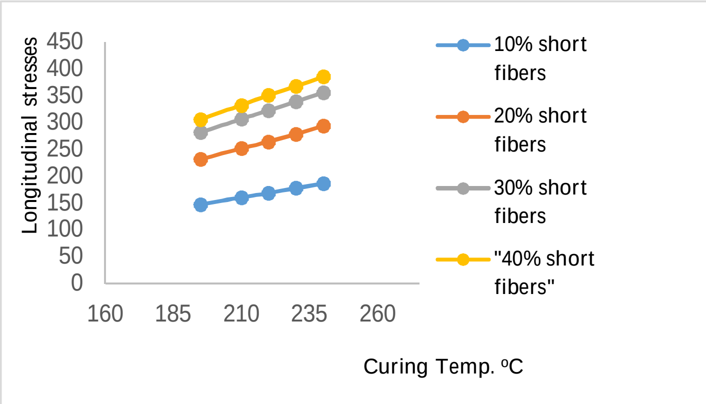

Fig.8: Effect of curing temp. on the longitudinal stresses (σx) of anti-symmetric stacking sequence.

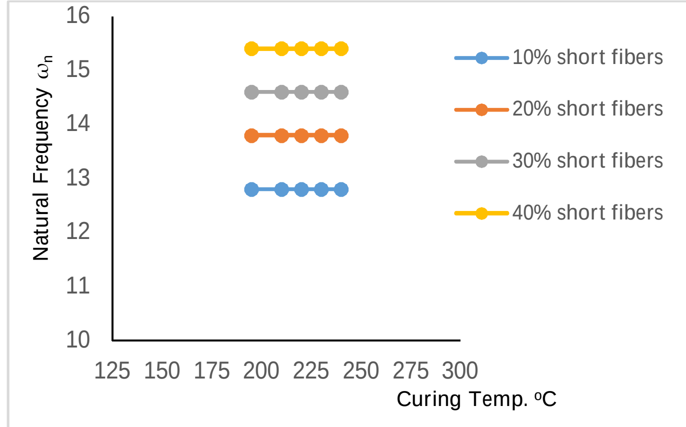

Fig.11: Effect of curing Temp. on the natural frequency ωn of an anti-symmetric stacking sequence.

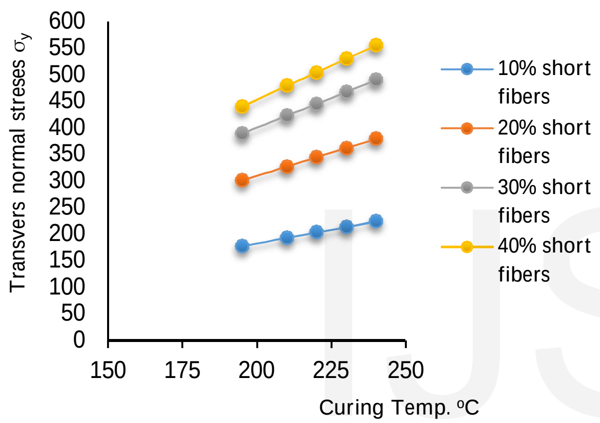

Fig.9: Effect of curing Temp. on the transverse normal stresses (σy) of anti-symmetric stacking sequence.

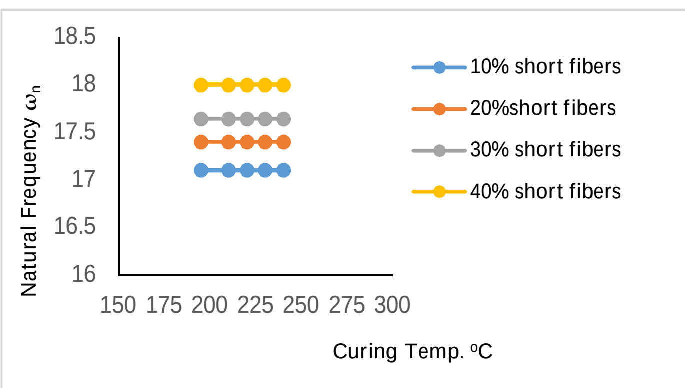

Fig.10: Effect of curing Temp. on the natural frequency ωn of a symmetric stacking sequence.

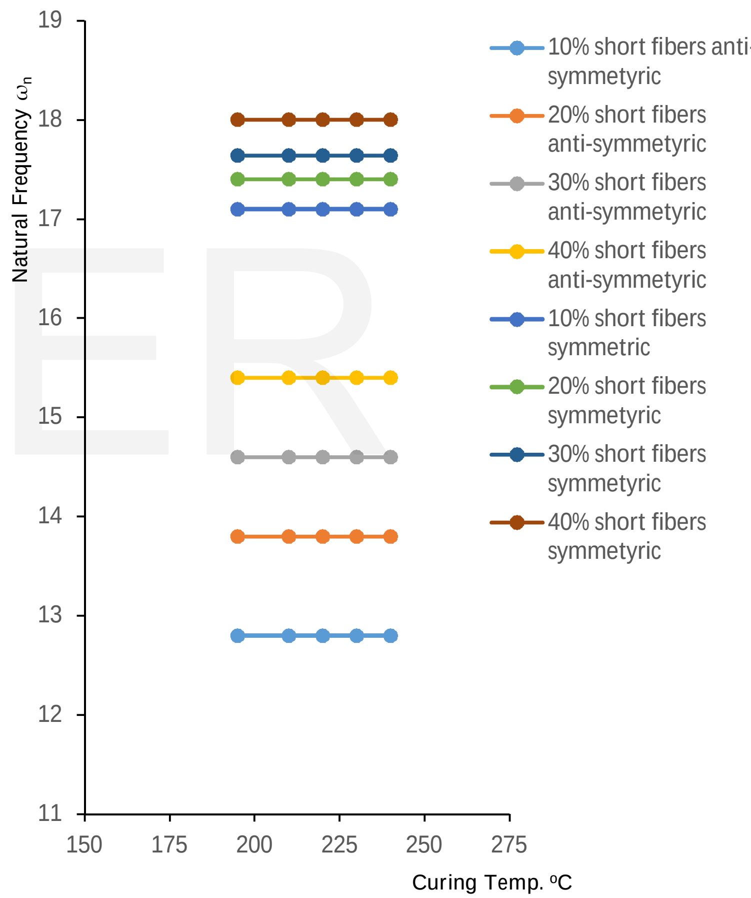

Fig.12: A comparison between effects of stacking sequence on the natural frequencies.

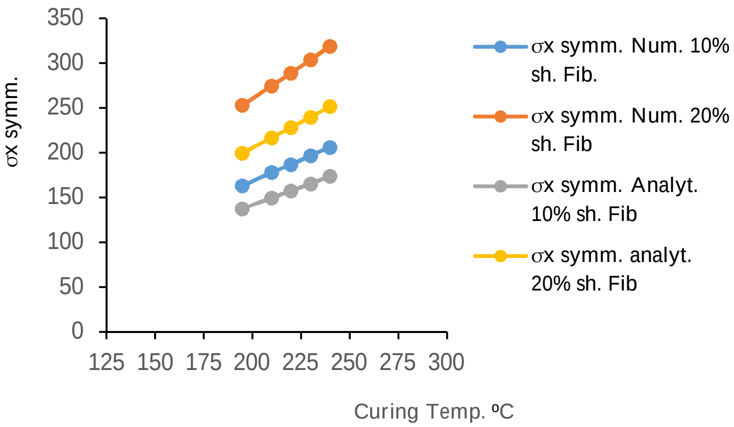

Fig.13: A comparison between the numerical and analytical results of longitudinal stresses induced.

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